GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

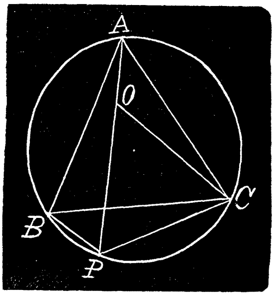
SOLUTIONS TO PROBLEMS.

19. Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.

If any point be taken in the circumference of a circle, and lines be drawn from it to the three angles of an inscribed equilateral triangle, prove that the middle line so drawn is equal to the sum of the other two.

I. Solution by CHARLES E. MYERS, Canton, Ohio; P. S. BERG, Apple Creek, Ohio; Professor F. E. MILLER, Westerville, Ohio; R. H. YOUNG, Sunbury, Pennsylvania.

Let ABC be an equilateral triangle inscribed in a circle, P any point in the circumference; and PB, PC and PA lines drawn from the point to the three angles.



On PA take $PO=PC$, and join OC. The angles OPC and OPB, being measured by equal arcs, are equal and constant $=60°$, and since $PO=PC$, the the triangle POC is equilateral and $OC=PC$.

In the triangles OCA and BCP we have $OC=PC$; $AC=BC$ and the angle ACO=angle BCP; therefore $AO=BP$. But $OP=PC$; therefore $BP+PC=PO+OA=PA$.

II. Solution by J. W. WATSON, Middle Creek, Ohio.

Let ABC be the inscribed equilateral triangle. P any point in the circumference. Join AP, CP and BP. We are to prove $AP=BP+CP$.

Put $a=$ a side of the triangle, $n=AP$, $z=BP$ and $s=CP$.

In the triangle APB, $a^2=n^2+z^2-nz$ since the angle APB is $60°$.

In the triangle CPB, $a^2=s^2+z^2+sz$ since the angle CPB is $120°$.

$\therefore n^2+z^2-nz=s^2+z^2+sz$, $n^2-s^2=sz+nz$ or $(n-s)(n+s)=z(n+s)$, $n-s=z$. $\therefore n=s+z$, or $AP=CP+BP$.

III. Solution by Professor J. R. BALDWIN, A. M., Davenport, Iowa, and T. A. SIMMONS, St. Mary's Kentucky.

Let P be any point in the circumference, ABC the inscribed equilateral triangle of which A and C lie next to P.

Draw AP and PC.

By a well known proposition $AB\times PC+AP\times BC=AC\times PB$, (1).

But $AB=BC=AC$, hence casting out the equal factor out of (1) we have $PC+AP=PB$. *Q. E. D.*

Also solved by *G. B. M. ZERR, J. H. DRUMMOND, M. A. GRUBER, J. K. ELLWOOD,* and *P. H. PHILBRICK.* ☞ Three other solutions *without names of contributors* were received.

21. Proposed by CHARLES E. MYERS, Canton, Ohio.

A cistern 6 feet in diameter contains 3 inches of water. If a cylinder, four